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Siebenmann-Type Cobordisms with Borders and Topology Changes by Quantum Tunneling

Vladimir N. Efremov[1]

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Siebenmann-type cobordisms are constructed to describe topology changes with the Seifert fibered homology spheres in in- and out-states. We study the problem of determining of topology-changing amplitudes for these quantum tunneling processes. The calculations are performed in the stationary phase approximation for Kodama wave functions. In this approximation the amplitudes are expressed in terms of Chern–Simons invariants of flat $SU(2)$-connections over the cobordism boundary components. The topology-change amplitudes found are factorized into the Kodama wave functions for the lens spaces. The results are compared with those for Fintushel–Stern-type cobordisms which have been previously investigated.

1. INTRODUCTION

The topology-change problem is one of the obstacles to constructing (3 + 1)-dimensional nonperturbative quantum gravity (Horowitz, 1991). The purpose of this article is to investigate a wide class of topology changes which are described by 4D Siebenmann-type cobordisms (Siebenmann, 1979). The spatial sections of these cobordisms are 3D Seifert fibered manifolds (Sf-manifolds) (Orlik, 1972). An Sf-manifold is a 3-dimensional manifold which is represented as union of pairwise disjoint loops (knots). We restrict our consideration to topology changes in an ensemble of Seifert fibered homology spheres (Sfh-spheres). They represent a direct generalization of the ordinary sphere S^3, but have some advantages; for example, links are naturally defined in the Sfh-spheres. [A link L is a finite collection of smooth, disjoint, oriented, simple closed curves (loops) $\Gamma = \{\gamma_i | i = 1, \ldots, m\}$ (Eisenbud and Neumann, 1985).] In nonperturbative quantum gravity links

[1] Departamento de Matematicas, Universidad de Guadalajara, Guadalajara, Jalisco, México; e-mail: vefremov@physics.fismat.udg.mx.

are used to define gauge-invariant variables (Rovelli and Smolin, 1990) and also to define a basis of states (Jacobson and Smolin, 1988) satisfying all constraint equations in connection representations. To consider topology changes it is necessary to use various natural operations over topological spaces and links such as connected sum, disjoint sum, and cabling, which can be used to construct new spaces and new links from initial ones. As demonstrated by Eisenbud and Neumann (1985), these operations are special cases of one which is called *splicing*. However, the splicing operation more probably results in links in Sfh-spheres rather than in S^3. It is therefore most convenient to utilize Sfh-spheres as the ambient spaces for links from the beginning. Using the techniques of Siebenmann's cobordism constructions, we demonstrate in Section 3 that topology changes with participation of Sfh-spheres ("maternal" universes) are accompanied by creation and annihilation of 3-spheres with two exceptional fibers ("baby" universes).

Our basic assumption is that all physical quantum states are represented as expansions with respect to Kodama wave functions (Kodama, 1990). Under this supposition the topology-changing amplitudes are constructed in connection representation. A stationary phase approximation is utilized, i.e., we restrict our consideration to the equivalence classes of flat connections on 3D sections of 4D cobordisms which describe the topology change. This situation naturally takes place in (2 + 1)-gravitation, as a consequence of the fact that the vacuum solutions of (2 + 1)-dimensional Einstein equations are flat Poincaré connections.

Moreover, there are different modifications of general relativity corresponding to exactly soluble diffeomorphism invariant $B \wedge F$-theories which were initiated by Horowitz (1989). The field equations in $B \wedge F$-theories require the connection to be both flat and torsion-free, just as in the (2 + 1)-dimensional case. The reduced phase space variables are the holonomies of flat connections for loops which form a basis of the first homotopy group.

In developing the analogy with (2 + 1)-gravity we use one more (2 + 1)-dimensional toy-model observation. It was demonstrated (see, e.g., Martin, 1989) that in 3D gravity it is possible to introduce a nontrivial dynamics by allowing conical singularities to represent point particles. Similarly, it might be useful to enrich the (3 + 1)-dimensional theory by admitting 1-dimensional exceptional orbits (fibers) corresponding to string structures. It is known (Eisenbud and Neumann, 1985) that the Sf-manifolds and the Seifert links provide an appropriate model of spatial sections possessing such exceptional orbits relative to the action of the group S^1 [$\simeq U(1)$]. We consider this type of manifold as admissible 3-dimensional section of Euclidean space-time cobordisms describing topology changes. In physical terms this model corresponds to the concept of a universe which is completely woven of closed strings (loops) (Ashtekar *et al.*, 1992).

Sections 2 presents important notations and definitions of basic concepts.

In Section 3 the Siebenmann-type cobordisms with borders are constructed. They contain some Euclidean-signature regions and therefore can be interpreted as space-time models of tunneling topology changes.

In Section 4 a certain type of boundary condition between Lorentzian and Euclidean signature regions is discussed.

In Section 5 topology-changing amplitudes are constructed in the connection representation on the basis of Kodama's wave functions for Siebenmann-type cobordisms. A stationary-phase approximation corresponding to the flat-connections contributions to topology-changing amplitudes is used. These results are compared with the ones obtained in the case of Fintushel–Stern cobordisms.

2. BASIC CONCEPTS: DEFINITIONS AND NOTATIONS

We refer to Neumann and Raymond (1978) and Scott (1983) for basic concepts and terminology about Sf-manifolds and particularly about Sfh-spheres. Moreover, to construct the Siebenmann-type cobordism we will utilize two topology operations, a *splicing* process for Sfh-spheres and a *splitting* process inverse to the first one, also known as a *vertical pinch* (Siebenmann, 1979; Rong, 1993).

In this paper a Seifert fibered manifold (Sf-manifold) will mean an oriented closed connected 3D manifold $M(\underline{a}) \equiv M(a_1, \ldots, a_n)$ admitting a pseudofree S^1-action. A pseudofree S^1-action is a smooth action such that it is free except for finitely many exceptional orbits (singular fibers) $S_1, \ldots, S_n$ with isotropy $Z_{a_1}, \ldots, Z_{a_n}$, respectively, where $a_1, \ldots, a_n \in Z$; $a_i > 1$, $i = 1, \ldots, n$. If

$$p\colon\quad M(\underline{a}) \to M(\underline{a})/S^1 = S^2(\underline{a}) \equiv S^2(a_1, \ldots, a_n) \tag{2.1}$$

is the Seifert fibration, then the base space $S^2(\underline{a})$ is an orbifold that possesses n exceptional cone points (with conical angles $2\pi/a_i$, $i = 1, \ldots, n$). We shall restrict our attention to the case where the genus g of the orbifold $S^2(\underline{a})$ is zero.

Let

$$TS_1 = (S^1 \times D^2)_1, \ldots, \quad TS_n = (S^1 \times D^2)_n$$

be disjoint tubular neighborhoods of exceptional fibers $S_1, \ldots, S_n$, and $M_0(\underline{a}) = M(\underline{a}) - TS_1 \sqcup \cdots \sqcup TS_n$ ($\sqcup$ is disjoint union). Since

$$p_0\colon\quad M_0(\underline{a}) \to M_0(\underline{a})/S^1 = S_0^2(\underline{a}) \tag{2.2}$$

[where $S_0^2(\underline{a}) = S^2 - D_1^2 \sqcup \cdots \sqcup D_n^2$, the disc D_i^2 being a neighborhood of i's cone point; when $n = 3$, $S_0^2(a_1, a_2, a_3)$ is a Thurston trinion] is an ordinary S^1-bundle over a connected surface with boundary, which admits a section surface $R \subset M_0(\underline{a})$, then $x_i = -\partial R \cap TS_i \subset \partial(TS_i)$ is a torus section curve (torus knot), $i = 1, \ldots, n$. If h is a typical fiber (regular orbit) of S^1-fibering p, then a curve $a_i x_i + b_i h$ is homological to zero (i.e., is a boundary) in TS_i. Therefore

$$\pi_1(M(\underline{a})) = \langle x_1, \ldots, x_n, h \mid h x_i h^{-1} x_i^{-1} = 1, x_i^{a_i} h^{b_i} = 1, x_1 \cdots x_n = 1 \rangle \tag{2.3}$$

is a fundamental group presentation of Sf-manifold $M(\underline{a})$ with unnormalized Seifert invariants (Neumann and Raymond, 1978)

$$(g = 0; (a_1, b_1), \ldots, (a_n, b_n)) \tag{2.4}$$

which satisfy $\gcd(a_i, b_i) = 1$, i.e., a_i and b_i are relatively prime for every i. If also $a_1, \ldots, a_n$ are pairwise relatively prime and satisfy

$$\sum_{i=1}^{n} b_i \sigma_i = 1, \qquad \text{where} \quad \sigma_i = a/a_i, \quad a = a_1 \cdots a_n \tag{2.5}$$

then the Sf-manifold $M(\underline{a})$ is called a *Seifert fibered homology sphere* (Sfh-sphere), i.e., this Sf-manifold has homology groups that are isomorphic to those of an ordinary 3-sphere S^3. In this case the integers b_i are defined, since we can take $b_i \sigma_i \equiv 1 \pmod{a_i}$ from (2.5). We use the standard notation $\Sigma(a_1, \ldots, a_n) = \Sigma(\underline{a})$ for the Sfh-sphere.

Strictly speaking, a link $L = (\Sigma(\underline{a}), L_1 \cup \cdots \cup L_m)$ is a pair consisting of an Sfh-sphere $\Sigma(\underline{a})$ and a collection of smooth, disjoint, simple closed curves $L_i \subset \Sigma(\underline{a})$. A Seifert link is a link L whose exterior $\Sigma(\underline{a}) - (TL_1 \cup \cdots \cup TL_m)$ admits a Seifert fibration (Eisenbud and Neumann, 1985).

We know that for a Sf-manifold to be a Sfh-sphere no less than three exceptional orbits are necessary. When $n = 1, 2$ a Sf-manifold is either a lens space $L(a, b)$ or an ordinary 3-dimensional sphere S^3 endowed with the structure of S^1-fibering which has the Seifert invariants (a_1, a_2) (Scott, 1983). In the last case, the Sf-manifold $M(a_1, a_2)$ is homeomorphic to S^3. Let us consider in more detail the construction of this Sf-manifold, since it is $M(a_1, a_2)$ that arises as one boundary component of the Siebenmann-type cobordism. Let TS_1 be a Seifert fibered solid torus such that a typical fiber h_1 (on the boundary ∂TS_1) is expressed by a topology standard longitude l_1 and a meridian m_1 as $h_1 = a_1 l_1 + a_2 m_1$. [The simple closed curve h_1 is a (a_1, a_2)-cable knot on ∂TS_1 (Eisenbud and Neumann, 1985).] Similarly, for the second solid torus TS_2 we have a typical fiber $h_2 = a_2 l_2 + a_1 m_2$. The Seifert structures of these solid tori are sewed smoothly along the boundaries ∂TS_1 and ∂TS_2

if $h_1 = h_2$. This condition leads to the following sewing rule: $l_1 = m_2$ and $l_2 = m_1$ for the solid tori. It is well known that the manifold obtained as a result of this sewing operation is homeomorphic to a sphere S^3. Thus we obtain the 3-dimensional sphere $M(a_1, a_2)$ with the Seifert structure (S^1-fibering). For more details we refer to Scott (1983).

It will be clear from Section 3 that the sewing operation, which has been defined for two Seifert fibered solid tori, is a special case of the general splicing operation. The latter plays a fundamental role in the process of construction of Siebenmann-type cobordisms (Siebenmann, 1979).

3. SPLITTING AND SPLICING OF Sfh-SPHERES

Let $M(\underline{a}_{1,n}) \equiv M(a_1, \ldots, a_n)$ be a Sf-manifold. Suppose that T is a separating vertical (consisting of fibers) torus in $M(\underline{a}_{1,n})$ so that

$$M(\underline{a}_{1,n}) = X(\underline{a}_{1,i}) \cup_T X(\underline{a}_{i+1,n}) \tag{3.1}$$

where

$$X(\underline{a}_{1,i}) = X(a_{j_1}, \ldots, a_{j_i}) \qquad \text{and} \qquad X(\underline{a}_{i+1,n}) = X(a_{j_{i+1}}, \ldots, a_{j_n})$$

are Seifert fibered manifolds with the boundaries

$$\partial X(\underline{a}_{1,i}) = -\partial X(\underline{a}_{i+1,n}) = T$$

The set $\{a_{j_1}, \ldots, a_{j_i}\}$ is nonempty subset of the set $\{a_1, \ldots, a_n\}$ such that $2 \leq i \leq n - 2$. Since the manifolds $X(\underline{a}_{1,i})$ and $X(\underline{a}_{i+1,n})$ inherit the Seifert fibered structures, this situation is a special case of the Seifert link structures (Eisenbud and Neumann, 1985). In other words, the torus T separates the Sf-manifold $M(\underline{a}_{1,n})$ such that i exceptional fibers are found in the Seifert fibered submanifold $X(\underline{a}_{1,i})$ and the other $(n - i)$ exceptional fibers are found in $X(\underline{a}_{i+1,n})$.

Let

$$M(\underline{a}_{0,i}) = X(\underline{a}_{1,i}) \cup_T TS_0 \tag{3.2}$$

where TS_0 is a solid torus whose meridian $m_0 \subset \partial(TS_0)$ is identified with a curve $a_0 x_0 + b_0 h$ in $\partial X(\underline{a}_{1,i})$. We utilize the following notations:

$$a_0 = a_{j_{i+1}} \cdots a_{j_n}; \qquad b_0 = a_0 \sum_{r=i+1}^{n} b_{j_r}/a_{j_r}; \qquad x_0 = R \cap \partial X(\underline{a}_{1,i})$$

x_0 is a section curve, and h is a typical fiber of the Sf-manifold $M(\underline{a}_{1,n})$.

It has been demonstrated (Siebenmann, 1979) that the space $M(\underline{a}_{0,i})$ is a Sf-manifold with $(i + 1)$ exceptional fibers $S_{j_0}, S_{j_1}, \ldots, S_{j_i}$ and Seifert invariants $\{(a_{j_s}, b_{j_s}) | s = 0, 1, \ldots, i\}$. Should the initial Sf-manifold $M(\underline{a}_{1,n})$ be a Sfh-sphere, then $M(\underline{a}_{0,i})$ would be one, too.

The analogous assertions are valid for the space

$$M(\underline{a}_{i+1,n+1}) = X(\underline{a}_{i+1,n}) \cup_T TS_{n+1} \tag{3.3}$$

where TS_{n+1} is a solid torus whose meridian $m_{n+1} \subset \partial(TS_{n+1})$ is identified with a curve $a_{n+1}x_{n+1} + b_{n+1}h$ in $\partial X(\underline{a}_{i+1,n})$. We utilize the same notations

$$a_{n+1} = a_{j_1} \cdots a_{j_i}; \qquad b_{n+1} = a_{n+1} \sum_{s=1}^{i} b_{j_s}/a_{j_s}; \qquad x_{n+1} = R \cap \partial X(\underline{a}_{i+1,n})$$

The operation which associates the Sf-manifolds $M(\underline{a}_{0,i})$ and $M(\underline{a}_{i+1,n+1})$ with the Sf-manifold $M(\underline{a}_{1,n})$ is called a *vertical pinch* (Rong, 1993).

It is important that the Seifert manifold Euler number $e = \Sigma_{i=1}^{n} b_i/a_i$ is conserved under this operation, because

$$e = \sum_{s=0}^{i} b_{j_s}/a_{j_s} = \sum_{r=i+1}^{n+1} b_{j_r}/a_{j_r}$$

The vertical pinch operation splits the Sfh-sphere $\Sigma(\underline{a}_{1,n})$ into two Sfh-spheres $\Sigma(\underline{a}_{0,i})$ and $\Sigma(\underline{a}_{i+1,n+1})$. Siebenmann (1979) constructed a cobordism

$$W(\underline{a}_{1,n}) \equiv W(a_1, \ldots, a_n)$$

which made this interpretation more explicit. We shall consider only the case when a starting Sf-manifold is a Sfh-sphere.

In our notations it is possible to define Siebenmann's cobordism as follows:

$$W(\underline{a}_{1,n}) = (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times [-1, 0] \cup (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [0, 1] \tag{3.4}$$

Observation 3.1. It is clear that Sfh-spheres $\Sigma(\underline{a}_{0,i})$ and $\Sigma(\underline{a}_{i+1,n+1})$ depend on the partition of the set $\underline{a} = \{a_1, \ldots, a_n\}$ into nonempty subsets $\{a_{j_1}, \ldots, a_{j_i}\}$ and $\underline{a} - \{a_{j_1}, \ldots, a_{j_i}\} = \{a_{j_{i+1}}, \ldots, a_{j_n}\}$, $2 \le i \le n - 2$. Therefore these Sfh-spheres and the cobordism $W(\underline{a}_{1,n})$ are more properly denoted as

$$\Sigma(\underline{a}_{0,i})\{j_1, \ldots, j_i\}, \qquad \Sigma(\underline{a}_{i+1,n+1})\{j_{i+1}, \ldots, j_n\}$$

and

$$W(\underline{a}_{1,n})\{j_1, \ldots, j_i\} \equiv W(\underline{a}_{1,n})\{j_{i+1}, \ldots, j_n\}$$

respectively. We will utilize the shortened notations when the information contained in the complementary indices is not important.

The cobordism (3.4) describes the topology change

$$\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1}) \to \Sigma(\underline{a}_{0,i}) \sqcup \Sigma(\underline{a}_{i+1,n+1}) \tag{3.5}$$

since it has the boundary

$$\partial W(\underline{a}_{1,n}) = \partial(W(\underline{a}_{1,n}))^{\text{in}} \sqcup \partial(W(\underline{a}_{1,n}))^{\text{out}} \tag{3.6}$$

where

$$\partial(W(\underline{a}_{1,n}))^{\text{in}} = (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times \{-1\}$$

and

$$\partial(W(\underline{a}_{1,n}))^{\text{out}} = (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times \{1\}$$

Siebenmann (1979) introduced a thickening $N(T)$ of the separating torus T to pull apart the singularities of the critical space section in $t = 0$; however, the creases at $\partial X(\underline{a}_{1,i}) \times \{0\}$ and $\partial X(\underline{a}_{i+1,n}) \times \{0\}$ remained and had to be smoothed out.

We propose another method to remove the singularities from the Siebenmann cobordism. To this end we observe first that the following surgery takes place in the Siebenmann cobordism at the instant when the cobordism parameter $t = 0$:

(1) The Sfh-sphere $\Sigma(\underline{a}_{1,n})$ breaks into two Sf-manifolds with borders

$$\Sigma(\underline{a}_{1,n}) \to X(\underline{a}_{1,i})) \cup X(\underline{a}_{i+1,n}) \tag{3.7}$$

and analogously

$$M(a_0, a_{n+1}) \to TS_0 \cup TS_{n+1} \tag{3.8}$$

(2) The formation (pasting) occurs at the same time $t = 0$. As a result two new Sfh-spheres

$$\Sigma(\underline{a}_{0,i}) = X(\underline{a}_{1,i}) \cup_T TS_0 \tag{3.9}$$

$$\Sigma(\underline{a}_{i+1,n+1}) = X(\underline{a}_{i+1,n}) \cup_T TS_{n+1} \tag{3.10}$$

arise (vertical pinch operation).

We suggest separating these two events: (1) bifurcations (3.7) and (3.8) will take place at $t = 0$ and (2) pasting together (3.9) and (3.10) will take place at $t = 1$. As a result we receive a *Siebenmann-type cobordism with a border* (Rourke and Sanderson, 1972)

$$\begin{aligned} W^{\text{bord}}(\underline{a}_{1,n}) = {} & (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \\ & \times [-1, 0] \cup (X(\underline{a}_{1,i})) \sqcup X(\underline{a}_{i+1,n}) \sqcup TS_0 \sqcup TS_{n+1}) \\ & \times [0, 1] \cup (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [1, 2] \end{aligned} \tag{3.11}$$

The boundary of this cobordism is divided into two parts:

(1) The *ordinary boundary*

$$\partial W^{\text{bord}}(\underline{a}_{1,n}) = \partial(W^{\text{bord}}(\underline{a}_{1,n}))^{\text{in}} \sqcup \partial(W^{\text{bord}}(\underline{a}_{1,n}))^{\text{out}} \tag{3.12}$$

where

$$\partial(W^{\text{bord}}(\underline{a}_{1,n}))^{\text{in}} = (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times \{-1\}$$

and

$$\partial(W^{\text{bord}}(\underline{a}_{1,n}))^{\text{out}} = (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times \{2\}$$

which is homeomorphic to (3.6).

(2) The *border*

$$\partial(W^{\text{bord}}(\underline{a}_{1,n}))^{\text{interm}} = (\partial X(\underline{a}_{1,i})) \sqcup \partial X(\underline{a}_{i+1,n}) \sqcup \partial TS_0 \sqcup \partial TS_{n+1}) \times [0, 1] \tag{3.13}$$

Now let us return to the Siebenmann cobordism (3.4) and consider an inverse oriented one,

$$-W(\underline{a}_{1,n}) = (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [-1, 0] \cup (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times [0, 1] \tag{3.4$'$}$$

This cobordism describes the inverse process

$$\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1}) \to \Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1}) \tag{3.5$'$}$$

with respect to the topology change (3.5).

At the critical level $t = 0$ two solid tori TS_0 and TS_{n+1} are separated from the Sfh-spheres $\Sigma(\underline{a}_{0,i}))$ and $\Sigma(\underline{a}_{i+1,n+1})$, respectively. At the same time the splicing process forms the Sfh-sphere

$$\Sigma(\underline{a}_{1,n}) = X(\underline{a}_{1,i}) \cup X(\underline{a}_{i+1,n}) \tag{3.14}$$

and the Sf-manifold

$$M(a_0, a_{n+1}) = TS_0 \cup_T TS_{n+1} \tag{3.15}$$

homeomorphic to S^3.

In the inverse cobordism

$$-W^{\text{bord}}(\underline{a}_{1,n}) = (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [-2, -1] \cup (X(\underline{a}_{1,i})) \sqcup X(\underline{a}_{i+1,n}) \sqcup TS_0 \sqcup TS_{n+1}) \times [-1, 0] \cup (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times [0, 1] \tag{3.11$'$}$$

these processes (splitting and splicing) are separated by an interval $[-1, 0]$ of the cobordism time.

From the physical point of view, the existence of the cobordism borders (3.13) indicates that some of the spacelike sections have torus borders $\partial X(\underline{a})$. These borders mark the limits of where we may probe with our measuring instruments. The idea is that border terms in the gravitational field action functional carry the significant part of the information (or all of it) about a removed part. The most important contributions in this direction are Hawking (1976), 't Hooft (1993), and Susskind (1994). Recently Smolin (1995) has developed the Hamilton method to describe the gravitational field in Ashtekar variables for the space-time manifold with borders of type $\partial\Sigma \times [0, 1]$. The main advantage of the method is the possibility to use the Chern–Simons theory in $(2 + 1)$ or 3 dimensions to describe a set of observables in nonperturbative quantum gravity in $(3 + 1)$ or 4 dimensions.

In the following sections we shall demonstrate that the border terms contribute significantly to the tunneling topology-change amplitude. The Kodama wave function gives the simplest way to describe topology transformations. Since we shall be working in the stationary phase approximation, we will restrict the connection to be flat over Sfh-spheres. The space of all flat connections over Sfh-spheres modulo gauge group transformations is described by Fintushel and Stern (1990) and Kirk and Klassen (1991).

4. SEWING TOGETHER EUCLIDEAN AND LORENTZIAN SIGNATURE REGIONS

Let us examine the topology-changing processes in the "quantum mechanics of topology" (quantum tunneling). In general these processes are classically forbidden since they are transitions through Euclidean-signature regions. We adopt the Kodama semiclassical approach (Kodama, 1990; Brügmann *et al.*, 1992). The wave function or transition amplitude is written as

$$\Psi(W_0) = C \exp\left(-\frac{1}{\lambda}\, CS(W_0)\right) \tag{4.1}$$

where

$$CS(W_0) = \frac{1}{8\pi^2}\int_{W_0} \mathrm{Tr}(F_A \wedge F_A) \tag{4.2}$$

is the Chern–Simons invariant of the 4-dimensional elementary cobordism $W_0 = M \times I$, $I = [0, 1]$, F being a curvature of a connection A. This amplitude is the known solution to all constraint equations in Ashtekar's connection representation. The wave function (4.1) is the unique solution in the quantum version of the Horowitz $B \wedge F$-theory (Horowitz, 1989). Thus we can obtain the Kodama quantum scheme starting from different nonequivalent classical theories.

Let A_t be a path of connection A_t, $t \in [0, 1]$, over the 3-manifold M. According to Kirk and Klassen (1992), this path determines a connection A on the elementary cobordism $M \times I$. We consider the $SU(2)$-connection only. Every $SU(2)$-bundle over the 3-manifold is trivial. The Chern–Simons invariant is a real-valued functional on the space of all connections on a trivial bundle. The Chern–Simons invariant is only well defined in $\mathbb{R}/\mathbb{Z}$ if we do not want to specify a trivialization. Let us choose a path A_t of connection from A_0 ($t = 0$) to A_1 ($t = 1$). Then

$$CS(W_0) = CS(A_1) - CS(A_0) = \frac{1}{8\pi^2} \int_{W_0} \mathrm{Tr}(F_A \wedge F_A) \tag{4.3}$$

In particular, if we choose A_0 to be a trivial connection (so the form $A_0 \equiv 0$), then we obtain another definition of the Chern–Simons invariant (Okonek, 1991)

$$CS(M^{\mathrm{E}}) = CS(A_1) = \frac{1}{8\pi^2} \int \epsilon^{abc}\, \mathrm{Tr}\left(A_a \partial_b A_c + \frac{2}{3} A_a A_b A_c\right) \tag{4.4}$$

where A_a is the $SU(2)$ spatial connection corresponding to $M^{\mathrm{E}} \times \{1\}$ (the superscript E means that W_0 is the Euclidean-signature cobordism).

Let W be a composite cobordism, for example, (3.4). In this case the Kodama wave function has to be generalized because of the multicomponent cobordism boundary (Dijkgraaf and Witten, 1990). We shall consider the Siebenmann-type cobordisms, such as (3.11) including not only Euclidean-signature regions, but also Lorentzian ones, which are sewed along a hypersurface (Sf-manifold) having trivial connection. To introduce the Lorentzian signature it is necessary to change the real cobordism parameter t to the imaginary one $\tau = it$ and to perform a Wick rotation of fields (Fujiwara *et al.*, 1992). Thus the Chern–Simons functional becomes

$$CS(M^{\mathrm{L}}) = iCS(A_1) = \frac{i}{8\pi^2} \int \epsilon^{abc}\, \mathrm{Tr}\left(A_a \partial_b A_c + \frac{2}{3} A_a A_b A_c\right) \tag{4.5}$$

at any boundary component M^{L} of a Lorentzian-signature region.

Therefore the generalized Kodama amplitude expression for the Siebenmann-type cobordism is

$$\Psi(W) = C \exp\left(-\frac{1}{\Lambda^{\mathrm{E}}} \sum_{k=1}^{K} \epsilon_k CS(M_k^{\mathrm{E}}) - \frac{1}{\Lambda^L} \sum_{p=1}^{P} \delta_m CS(M_m^{\mathrm{L}})\right) \tag{4.6}$$

where M_k^E, $k = 1, \ldots, K$, are boundary (border) components of the Euclidean-signature regions; M_p^L, $p = 1, \ldots, P$, are boundary components of the Lorentzian-signature regions; Λ^E is the Euclidean coupling constant (cosmology constant); Λ^L is the Lorentzian coupling constant, or $k = [1/\Lambda^L]$ is the level of Chern–Simons theory [k should be an integer; see, e.g., Dijkgraaf and Witten (1990)]; and $\epsilon_k = \pm 1$; $\delta_m = \pm 2\pi$ (Brügmann *et al.*, 1992; Smolin, 1995).

Now we shall discuss what sort of boundary conditions must be imposed at the boundary $M_{\text{bound}}^L = M_{\text{bound}}^E$ between Lorentzian- and Euclidean-signature regions. Let the wave function (4.6) be continuous at the boundary $M_{\text{bound}}^L = M_{\text{bound}}^E$; then

$$CS(M_{\text{bound}}^L) = CS(M_{\text{bound}}^E) \tag{4.7}$$

However, the expressions (4.4) and (4.5) show that $CS(M_{\text{bound}}^L) = iCS(A)$ is purely imaginary, while $CS(M_{\text{bound}}^E) = CS(A)$ is real. It follows that the connection is trivial at the boundary $M_{\text{bound}}^L = M_{\text{bound}}^E$, i.e.,

$$CS(M_{\text{bound}}^L) = CS(M_{\text{bound}}^E) = 0 \tag{4.8}$$

Similar results were obtained in Halliwell and Hartle (1990) and Fujiwara *et al.* (1992).

This boundary condition gives the possibility to construct topology-change amplitude for the cobordisms (3.11) and (3.11′) if we restrict ourselves only to the flat-connection contributions on the cobordism spacelike sections $X \times \{t\}$, which have nontrivial borders $\partial X \times \{t\} \neq \emptyset$. For Siebenmann-type cobordisms with borders the result is nontrivial precisely due to the border contributions.

5. FLAT-CONNECTION CONTRIBUTIONS TO THE TOPOLOGY-CHANGING AMPLITUDES

We shall consider connection paths A_t, i.e., the connection over the elementary constituent parts $M_k^E \times I$ and $M_p^L \times I$ of the cobordism $W^{\text{bord}}(\underline{a}_{1,n})$. Connections over the initial and final hypersurfaces of the elementary cobordism are assumed to be always flat. It is easy to demonstrate that this restriction corresponds to the stationary-phase approximation for each multiplicative component

$$\Psi(M) = C \exp\left(-\frac{\mu}{\lambda} CS(A)\right) \tag{5.1}$$

of the cobordism wave function (A is the connection over M; $\mu = 1$ if $M = M_k^{\mathrm{E}}$ and $\mu = 2\pi i$ if $M = M_p^{\mathrm{L}}$). The relation

$$\frac{\delta}{\delta A_a^i} \Psi(M) = \frac{\mu}{4\pi^2\lambda} \epsilon^{abc} F_{bc}^i \Psi(M) \tag{5.2}$$

demonstrates that the flat connections ($F_{ab}^i = 0$) are critical points of the Chern–Simons functional [as a Morse function on the orbit space of connections over M modulo gauge invariance (Okonek, 1991)]. Consequently in the connection representation the flat connections are stationary points of the wave function's phase.

Now we turn to calculate the amplitude of the topology change (3.5), which is described by the cobordism $W^{\mathrm{bord}}(\underline{a}_{1,n})$, (3.11). In this case the initial conditions are fixed at the hypersurfaces $\Sigma(\underline{a}_{1,n}) \times \{-1\}$ and $M(a_0, a_{n+1})) \times \{-1\}$. The cobordism constituent parts with the Lorentzian signature are

$$(\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times [-1, 0] \tag{5.3}$$

The Euclidean-signature component parts of $W^{\mathrm{bord}}(\underline{a}_{1,n})$ are

$$(X(\underline{a}_{1,i})) \sqcup X(\underline{a}_{i+1,n}) \sqcup TS_0 \sqcup TS_{n+1}) \times [0, 1] \tag{5.4}$$

and

$$(\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [1, 2] \tag{5.5}$$

Accordingly, the boundaries between the Lorentzian- and Euclidean-signature regions are

$$(\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \times \{0\}, \qquad (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times \{2\} \tag{5.6}$$

These boundaries carry the trivial connections according to the condition (4.8). Moreover, the manifold $M(a_0, a_{n+1})$ is homeomorphic to S^3; therefore each flat connection over $M(a_0, a_{n+1})$ is trivial. We suppose that any path A_t on M (a_0, a_{n+1}) consists of trivial connections only.

We recall several key observations about the flat $SU(2)$-connections over a Sf-manifold M. The holonomy defines a homeomorphism between the space $R(M)$ of flat connection modulo the gauge group and the space $\hat{R}(M)$ of conjugancy classes of representations of the fundamental group $\pi_1(M)$ into $SU(2)$. If $M = \Sigma(\underline{a})$ is a Sfh-sphere, then two flat connections A and A' which lie on the same component of the space $R(\Sigma(\underline{a}))$ have the same Chern–Simons invariant $CS(A) \equiv CS(A')$ mod 1.

Removing a solid torus TS (a torus neighborhood of a fiber S) from a Sfh-sphere increases the dimension of the flat connection space: dim $R(X(\underline{a}))$ > dim $R(\Sigma(\underline{a}))$, where $X(\underline{a}) = \Sigma(\underline{a}) - TS$. In the space $R(X(\underline{a}))$ a piecewise smooth path A_t exists joining the flat connections A_0 and A_1 which lie in different components of the space $R(\Sigma(\underline{a}))$. Kirk and Klassen calculated the difference of Chern–Simons invariants $CS(A_1) - CS(A_2)$ for $\Sigma(\underline{a})$ as a function of the path joining the restrictions of these connections to $X(\underline{a})$. We use their results to calculate the contribution of the cobordism border (3.13) to the amplitude (4.6).

For a Sfh-sphere $\Sigma(\underline{a}) = \Sigma(a_1, \ldots, a_n)$ the set of connection components of $R(\Sigma(\underline{a}))$ is in one-to-one correspondence with the set of admissible collection of the rotation numbers $(\underline{l}) = (l_1, \ldots, l_n)$, which completely specify the gauge class of flat connections on $\Sigma(\underline{a})$. In other words, the admissible collection of rotation numbers contains the total information about the class α of irreducible representations of $\pi_1(\Sigma(\underline{a}))$ in $SU(2)$. Thus $CS(A) = CS(\underline{l})$.

The calculation schemes for the admissible collections $(l_1, \ldots, l_n)$ are developed by Fintushel and Stern (1990) and Kirk and Klassen (1991). In particular, the admissible collection must satisfy the following conditions:

$$\begin{aligned} &l_i \text{ is even if } b_i \text{ is even or } \alpha(h) = +1 \\ &l_i \text{ is odd if } b_i \text{ is odd and } \alpha(h) = -1 \end{aligned} \tag{5.7}$$

where b_i are defined by $b_i\sigma_i \equiv 1 \pmod{a_i}$ and $\alpha(h)$ is the image of the generator h of the center of the Sfh-sphere fundamental group $\pi_1(\Sigma(\underline{a}))$ [see (2.3)].

We shall utilize the Fintushel–Stern expression for Chern–Simons invariants

$$CS(A) = CS(\underline{l}) \equiv e_{1,n}^2/4a \pmod{1} \tag{5.8}$$

where $e_{1,n} = \sum_{i=1}^{n} l_i\sigma_i$ and A is a flat connection on the Sfh-sphere $\Sigma(\underline{a}_{1,n})$.

Kirk and Klassen (1990) developed a method which gives us the possibility to calculate the Chern–Simons invariant of the border (3.13),

$$CS((\partial W)^{\text{bord}}) = (e'_{1,n})^2/4a + (\delta/4)(l'_0 + l'_{n+1}) \tag{5.9}$$

where $\delta = 0$ if $\alpha(h) = 1$; $\delta = 1$ if $\alpha(h) = -1$; $e'_{1,n} = \sum_{i=1}^{n} l'_i\sigma_i$, and $(\underline{l}'_{0,i}) = (l'_0, l'_1, \ldots, l'_i)$ and $(\underline{l}'_{i+1,n+1}) = (l'_{i+1}, \ldots, l'_n, l'_{n+1})$ are the rotation number admissible collections, which define the flat connections over the $\Sigma(\underline{a}_{0,i})$ and $\Sigma(\underline{a}_{i+1,n+1})$, respectively.

It is important to observe that the three-dimensional Chern–Simons theory acts at the borders of Siebenmann-type cobordisms (Smolin, 1995). The cobordism parameter t (time) plays the role of the third dimension on the border components (3.13).

Thus the topology-changing amplitude is obtained simply by substitution of the corresponding Chern–Simons invariants into the expression (4.6),

$$\Psi(W^{\mathrm{bord}}(\underline{a}_{1,n})) = c(\underline{a}_{1,n}) \sum_{(\underline{l}'_{0,i}),(\underline{l}'_{i+1,n+1})} \exp\left(\frac{\pi i e_{1,n}^{2}}{2a\Lambda^{L}}\right) \times \exp\left[-\frac{1}{4\Lambda^{\mathrm{E}}}\left(\frac{(e'_{1,n})^{2}}{a} + \delta(l'_0 + l'_{n+1})\right)\right] \qquad (5.10)$$

where the summation is taken over the admissible collections $(\underline{l}'_{0,i})$ and $(\underline{l}'_{i+1,n+1})$, which determine the flat connections over $\Sigma(\underline{a}_{0,i}) \times \{1\}$ and $\Sigma(\underline{a}_{i+1,n+1}) \times \{1\}$, respectively (i.e., summation is performed over all flat connections in the intermediate state at $t = 1$). Furthermore, $c(\underline{a}_{1,n})$ is a constant depending apparently on the boundary volume-factors (Fujiwara *et al.*, 1992).

Observation 5.1. The cobordism $W^{\mathrm{bord}}(\underline{a}_{1,n})$ depends on the partition of the set $\underline{a} = \{a_1, \ldots, a_n\}$ into nonempty subsets $\{a_{j_1}, \ldots, a_{j_i}\}$ and $\{a_{j_{i+1}}, \ldots, a_{j_n}\}$ (see Observation 3.1). Thus not only are the Chern–Simons invariants $CS(\Sigma(\underline{a}_{0,i}))$ and $CS(\Sigma(\underline{a}_{i+1,n+1}))$ defined by this partition, but so are the coefficients $c(\underline{a}_{1,n})$. If the partition is not fixed by physical conditions, one must sum expressions of type (5.10) over all these unordered partitions.

Observation 5.2. Using our previous results (Efremov, 1996), we can demonstrate that the wave function (5.10) is factorized to the lens space wave functions

$$\Psi(W^{\mathrm{bord}}(\underline{a}_{1,n})) = c(\underline{a}_{1,n}) \sum_{(\underline{l}'_{0,i}),(\underline{l}'_{i+1,n+1})} \prod_{k=1}^{n} \exp\left(\frac{2\pi i}{\Lambda^{\mathrm{L}}}\, CS(L(a_k, b_k)\right) \times \exp\left(-\frac{1}{\Lambda^{\mathrm{E}}}\, CS'(L(a_k, b_k)\right) \exp\left(-\frac{\delta(l'_0 + l'_{n+1})}{4\Lambda^{\mathrm{E}}}\right) \qquad (5.11)$$

where

$$CS(L(a_k, b_k) = \frac{(l_i\sigma_i)^2}{4a}, \qquad CS'(L(a_k, b_k) = \frac{(l'_i\sigma_i)^2}{4a}$$

(Kirk and Klassen, 1990).

In the case of the Fintushel–Stern cobordism (Efremov, 1996) the rotation number collection was transferred from an initial hypersurface to a final one through the set of lens spaces $L(a_k, b_k)$, $k = 1, 2, \ldots, n$. Therefore the equalities $l_k = l'_k$, $k = 1, 2, \ldots, n$, were satisfied. In the Siebenmann-type cobordism with the trivial connection in an intermediate state (at $t = 0$) the admissible rotation number collections in the initial state and those in the final state are independent. Thus it should be summed over all admissible rotation number collections of the final state.

Similar arguments for the inverse cobordism (3.11′) lead to the other amplitude

$$\Psi(-W^{\text{bord}}(\underline{a}_{1,n}))$$

$$= c(\underline{a}_{1,n}) \sum_{(\underline{l}'_{1,n})} \exp\left[\frac{\pi i}{2a\Lambda^{\text{L}}}(e_{0,i}^2 + e_{i+1,n+1}^2)\right]$$

$$\times \exp\left[-\frac{1}{4\Lambda^{\text{E}}}\left(\frac{(e'_{1,n})^2}{a} + \delta(l'_0 + l'_{n+1})\right)\right] \tag{5.10′}$$

which describes the topology change (3.5′). In this case the initial state is fixed by the specification of connections over the Sfn-spheres $\Sigma(\underline{a}_{0,i}) \times \{-2\}$ and $\Sigma(\underline{a}_{i+1,n+1}) \times \{-2\}$, and summation is taken over all admissible collections $(\underline{l}'_{1,n})$ which fix the flat connection over $\Sigma(\underline{a}_{1,n}) \times \{0\}$ (intermediate state), while the rotation numbers l'_0 and l'_{n+1} are determined uniquely:

$$l'_0 = \sum_{j=i+1}^{n} \frac{l'_j}{a_j} a_0, \qquad a_0 = a_{i+1} \cdots a_n$$

$$l'_{n+1} = \sum_{j=1}^{i} \frac{l'_j}{a_j} a_{n+1}, \qquad a_{n+1} = a_1 \cdots a_i$$

Trivial connections on the cobordism $-W^{\text{bord}}(\underline{a}_{1,n})$ are defined over the Sfh-spheres $\Sigma(\underline{a}_{0,i}) \times \{-1\}$, $\Sigma(\underline{a}_{i+1,n+1}) \times \{-1\}$, and $\Sigma(\underline{a}_{1,n}) \times \{1\}$ and in the regions $M(a_0, a_{n+1}) \times [0, 1]$ and $(TS_0 \sqcup TS_{n+1}) \times [-1, 0]$.

Observation 5.3. In the case under consideration the initial state fixes the partition of the set $\{\underline{a}\}$ into two subsets (see Observation 3.1). Thus to obtain the total process amplitude, it should not be summed over different partitions as in Observation 5.1.

It is interesting to compare the amplitudes (5.10) and (5.10′) with the corresponding wave functions describing the topology changes (3.5) and (3.5′), but by means of the cobordisms (3.4) and (3.4′), respectively. The latter conform to "tunneling" through an infinitely thin Euclidean-signature layer in a neighborhood of the critical level

$$[X(\underline{a}_{1,i} \cup X(\underline{a}_{i+1,n}) \cup TS_0 \cup TS_{n+1}] \times \{0\} \tag{5.12}$$

To begin with we consider a 2ϵ-neighborhood of the critical level (bilateral ϵ-collar)

$$\begin{aligned}\textbf{(coll)} &= [X(\underline{a}_{1,i} \cup X(\underline{a}_{i+1,n}) \cup TS_0 \cup TS_{n+1}] \times [-\epsilon, \epsilon] \\ &= (\Sigma(\underline{a}_{1,n}) \sqcup M(a_0, a_{n+1})) \\ &\quad \times [-\epsilon, 0] \cup (\Sigma(\underline{a}_{0,i})) \sqcup \Sigma(\underline{a}_{i+1,n+1})) \times [0, \epsilon] \qquad (5.13)\end{aligned}$$

This is the Euclidean-signature region. The Lorentzian-signature regions are sewed at the boundary of the bilateral ϵ-collar. Therefore the trivial connection corresponds to the boundary ∂(**coll**). When $\epsilon \to 0$ we obtain the trivial connection level (5.12).

Then the wave function describing the tunneling through an infinitely thin Euclidean-signature layer is

$$\Psi(W^{\text{bord}}(\underline{a}_{1,n})) = c(\underline{a}_{1,n}) \sum_{(\underline{l}_{0,i}')(\underline{l}_{0,i}')} \exp\left[\frac{\pi i}{2\Lambda^{\text{L}} a}((e_{0,i}')^2 + (e_{i+1,n+1}')^2 - e_{1,n}^2)\right] \qquad (5.14)$$

The initial conditions are defined on the Sfh-sphere $(\Sigma(\underline{a}_{1,n}))$. Summation is performed over all admissible rotation number collections, which fix the flat connections over out-manifolds $(\Sigma(\underline{a}_{0,i}))$ and $\Sigma(\underline{a}_{i+1,n+1}))$. The inverse topology-changing process is characterized by the amplitude

$$\Psi(-W^{\text{bord}}(\underline{a}_{1,n})) = c(\underline{a}_{1,n}) \sum_{(\underline{l}_{1,n}')} \exp\left[\frac{\pi i}{2\Lambda^{\text{L}} a}((e_{0,i})^2 + (e_{i+1,n+1})^2 - (e_{1,n}')^2)\right] \qquad (5.14')$$

where summation takes place over all admissible collections $(\underline{l}_{1,n}')$ defining the flat connections on the final Sfh-sphere. The other component of the out-manifold $M(a_0, a_{n+1})$ carries the trivial connection as always. Analogous observations are valid for factorization of the amplitudes (5.14) and (5.14′) as well as for (5.10) and (5.10′). But in the amplitudes (5.14) and (5.14′) exponential decay terms are absent, since Euclidean-signature layers are finitely thin.

6. DISCUSSION AND CONCLUSION

In this paper the following three principal results may be emphasized.

1. The simplest examples of Siebenmann-type cobordisms with borders describing tunneling-topology changes through Euclidean-signature regions are constructed. These cobordisms differ from the Fintushel–Stern ones since

there are no lens spaces in the initial, intermediate, and final states. However, the Siebenmann-type cobordisms contain in the in- or out-state Sf-manifolds with two exceptional fibers which are homeomorphic to S^3.

2. Topology-change amplitudes for the Siebenmann-type cobordisms are evaluated. Just as in Efremov (1996), the stationary-phase approximation is used, but the trivial connection assumption for intermediate states is new. This supposition allows us to sew the Lorentzian- and Euclidean-signature regions such that wave functions are continuous on their boundary.

3. All wave functions in the stationary-phase approximation are expressed in terms of Chern–Simons invariants of a flat $SU(2)$-connection over Sfh-spheres and are factorized into wave functions expressed in terms of Chern–Simons invariants of the appropriate $SU(2)$-bundles over lens spaces. The tunneling-topology-changing amplitudes significantly differ from those obtained for the Fintushel–Stern cobordisms (Efremov, 1996). In the case of Fintushel–Stern cobordisms the flat-connection information is transferred partially from the initial Sfh-spheres to the final ones through the set of lens spaces. In our approach to Siebenmann-type cobordisms all the information about the connections is lost by the transition through hypersurfaces with the trivial connections. This is stipulated by the severe conditions of trivial connection over the boundaries between Euclidean- and Lorentzian-signature regions. But these are the natural conditions for wave functions to be continuous and they give us the possibility to apply the Kirk–Klassen method to evaluate the Chern–Simons invariants of flat $SU(2)$-connections over cobordism borders. The cobordism parameter t "time" serves as one (the third) dimension on the border. Thus at the border a three-dimensional Euclidean Chern–Simons theory does work (Smolin, 1995), which is exactly equivalent to Einstein's gravitation in three dimensions (Witten, 1988), which leads to flat connections as a consequence of the vacuum 3D Einstein equations. The very border-component contributions lead to the exponential decay terms in the wave functions (5.10) and (5.10′).

Our method permits us to evaluate the topological-change amplitudes for a more complicated process,

$$\bigsqcup_{i=1}^{n-2} \Sigma(a_1^i, a_{i+1}, a_{i+2}^n) \to \Sigma(\underline{a}_{1,n}) \bigsqcup_{i=2}^{n-2} M(a_1^i, a_{i+1}^n) \tag{6.1}$$

(where $a_1^i = a_1 \cdots a_i$, $a_{i+1}^n = a_{i+1} \cdots a_n$), as in the case of Fintushel–Stern cobordisms. This topology change can be interpreted as the creation of a universe $\Sigma(\underline{a}_{1,n})$ out of the simplest Sfh-spheres having the minimal number ($k = 3$) of exceptional fibers. The expression (6.1) may be considered as a three-dimensional analog of the Thurston *trinion decomposition* of a Sfh-sphere (Smolin, 1995; Crane, 1991) [see the explanation after the expression

(2.2)]. Thus the inverse process describes the decay of a Sfh-sphere into three-dimensional trinions, i.e., into Sfh-spheres with three exceptional fibers. These tunneling changes are accompanied by the creation or annihilation of Sf-manifolds $M(a_1^i, a_{i+1}^n)$ homeomorphic to S^3.

From our point of view the study of these topology changes may shed light on the problem of fixing fundamental constants (Weinberg, 1989; Klevanov *et al.*, 1989; Efremov, 1996).

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